Improved Complement for Two-Way Alternating Automata

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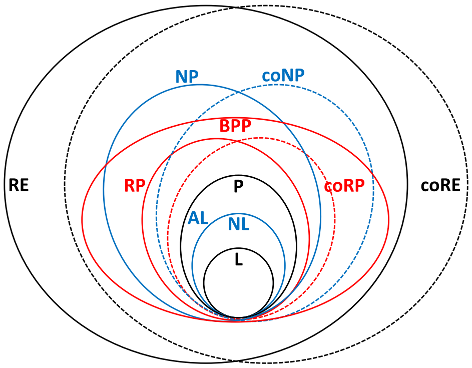
Motivation. Complementation of counter-less computing devices

A classic task in *Computational Complexity Theory* is to ***complement*** a computing device of a certain type:

Q1 Given device M of type X, is there device M' of the same type X which accepts exactly the inputs not accepted by M?

Often, "type X" is ***Turing machines*** (TM) of a certain ***mode*** which operate under certain time and/or space ***bounds***.



Depending on the mode and bound, the answer to (Q1) can be anywhere from trivial, e.g.:

1. ***deterministic polynomial-time*** (**P=coP**)
2. ***2-sided probabilistic polynomial-time*** (**BPP=coBPP**)

…to (non-trivial, but still) easy, e.g.:

3. ***deterministic logarithmic-space*** (**L=coL**)
4. ***alternating logarithmic-space*** (**AL=coAL**)

…to highly non-trivial, e.g.:

5. ***deterministic unbounded*** (**RE≠coRE**)
6. ***nondeterministic logarithmic-space*** (**NL=coNL**)

…to still unknown, e.g.:

7. ***nondeterministic polynomial-time*** (**NP=coNP** ?)
8. ***1-sided probabilistic polynomial-time*** (**RP=coRP** ?)

In some of these answers (e.g., **AL** = **coAL**), M' works correctly only if all computation branches of M are ***halting***. This we can assume if M can store a ***polynomially-large counter***; i.e., it has ***logarithmic space***.

What if such space is not available in X? I.e., what if M and M' work in ***sub-logarithmic space*** and are thus "***counter-less***"? For example:

Q2 Given alternating TM M of space $O(\log\log n)$, is there TM M' of the same kind which accepts exactly the inputs not accepted by M?

This (**ALL** = **coALL**) was answered affirmatively in [1], by a construction that works even if the space is 0, and thus M and M' are ***two-way alternating finite automata*** (2AFA).

However, it was not clear how many more states M' had, making the next question interesting:

Q3 Given 2AFA M with poly(h) states is there 2AFA M' with poly(h) states which accepts exactly the inputs not accepted by M?

Question. Complementation of small 2AFAs

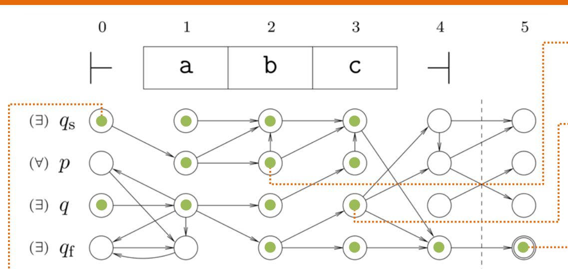
A **2AFA** $M = (S, \Sigma, \delta, q_s, q_f, U)$ is a tuple of the following:

- a set of ***states*** S
- an ***input alphabet*** Σ
- a ***start state*** $q_s \in S$
- a ***final state*** $q_f \in S$
- some ***universal*** states $U \subseteq S$ the rest in $S \setminus U$ are ***existential***
- a ***transition relation*** $\delta \subseteq S \times \Sigma \times S \times \{\text{L,S,R}\}$

The ***language*** of M is the set of all accepted inputs: $L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$

On input w of length n, a ***configuration*** is any state-position pair (q, i). It is ***accepting*** if:

- …q is *universal* and *all* out-neighbors are accepting; or
- …q is *existential* and *some* out-neighbor is accepting; or
- …(q, i) is exactly the ***accepting terminal configuration*** $(q_f, n+2)$.



M accepts w if the ***start configuration*** $(q_s, 0)$ is accepting.

M' complements M if, for all w: M' accepts $w \Leftrightarrow M$ does not accept w

Q3 Given 2AFA M with s states, does there exist 2AFA M' that complements M and has only $O(s^c)$ states, where c is some constant (independent of s and n)?

2A $\stackrel{?}{=}$ co2A

Background. 2A=co2A

An affirmative answer appeared in [2]. (We helped fix an oversight [3].) The construction lead to $c = 7$ and was quite involved, making use of an intermediate device.

M — 2AFA
- *s states*
- *accepts $L := L(M)$*

→

$\widetilde{M}$ — *deterministic linear bounded automaton*
- *checks that its input is a description $\mathcal{H}(w)$ of the configuration graph of M on w & the start configuration is accepting*
- *$O(s)$ work symbols*
- *$O(s^2)$ visits/cell*
- *$O(s^2)$ states*
- *accepts $\mathcal{H}(L)$*

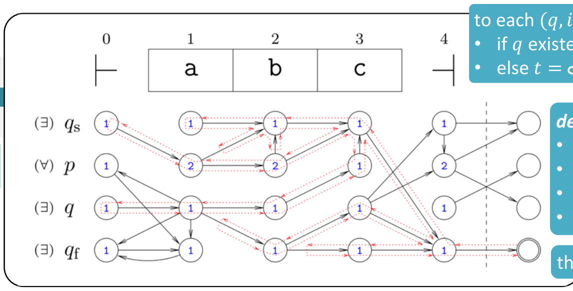
→

M' — 2AFA
- *checks that $\widetilde{M}$ rejects $\mathcal{H}(w)$*
- *$O(s^7)$ states*
- *accepts $\bar{L}$*

Contribution. A simpler and cheaper construction

We propose a new construction [4]. It is both simpler, as it eliminates the intermediate device; and cheaper, as it leads to $c = 6$.

M — 2AFA
- *s states*
- *accepts $L := L(M)$*



to each (q, i) assign a ***threshold*** t:
- if q existential then $t = 1$
- else $t =$ `out-degree`(q, i)

depth-first search the graph of M on w:
- start from $(q_f, n+2)$
- follow the arrows ***backwards***
- on each visit to a (q, i), decrement its t
- go to in-neighbords only after $t = 0$

this ***backward tour*** visits an existential (q, i) iff (q, i) is accepting

M' — 2AFA
- *checks that the backward tour does not visit $(q_s, 0)$*
- *$O(s^6)$ states*
- *accepts $\bar{L}$*

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References

[1] V. Geffert. *Alternating space is closed under complement and other simulations for sublogarithmic space*. Information and Computation, 253(1):163-178, 2017.

[2] V. Geffert. *Complement for two-way alternating automata*. Proceedings of Computer Science Theory and Applications in Russia (CSR), LNCS 10846, pp. 132–44, Springer, 2018.

[3] V. Geffert, C. Kapoutsis, M. Zakzok. *Complement for two-way alternating automata*. Acta Informatica, 2020. https://doi.org/10.1007/s00236-020-00373-8

[4] V. Geffert, C. Kapoutsis, M. Zakzok. *Improved complement for two-way alternating automata*. In preparation.